



NATIONAL SENIOR CERTIFICATE EXAMINATION
NOVEMBER 2020

TECHNICAL MATHEMATICS: PAPER II
MARKING GUIDELINES

Time: 3 hours

150 marks

These marking guidelines are prepared for use by examiners and sub-examiners, all of whom are required to attend a standardisation meeting to ensure that the guidelines are consistently interpreted and applied in the marking of candidates' scripts.

The IEB will not enter into any discussions or correspondence about any marking guidelines. It is acknowledged that there may be different views about some matters of emphasis or detail in the guidelines. It is also recognised that, without the benefit of attendance at a standardisation meeting, there may be different interpretations of the application of the marking guidelines.

QUESTION 1

$$1.1 \quad m_{AB} = \frac{y_B - y_A}{x_B - x_A}$$

$$m_{AB} = \frac{17-5}{12-3} = \frac{4}{3} \quad \checkmark^a \text{ substitute into correct formula}$$

$\checkmark^{ca}$ simplification

(2)

$$1.2 \quad m_{AB} \times m_{BC} = -1$$

$$\frac{4}{3} \times m_{BC} = -1$$

$$m_{BC} = \frac{-3}{4} \quad \checkmark^{ca} \text{ simplification}$$

$$m_{BC} = \frac{y_B - y_C}{x_B - x_C} = \frac{-3}{4}$$

$$\frac{17-20}{12-k} = \frac{-3}{4} \quad \checkmark^a \text{ substitute into correct formula}$$

$$4(17-20) = -3(12-k) \quad \checkmark^{ca} \text{ simplify}$$

$$-12 = -3(12-k)$$

$$4 = 12 - k$$

$$k = 8 \quad \checkmark^{ca} \text{ simplification}$$

(4)

$$1.3 \quad AB = \sqrt{(x_B - x_A)^2 + (y_B - y_A)^2}$$

$$= \sqrt{(12-3)^2 + (17-5)^2} \quad \checkmark^a \text{ substitute into correct formula}$$

$$= \sqrt{225}$$

$$= 15 \text{ units} \quad \checkmark^{ca} \text{ answer}$$

$$AB^2 + BC^2 = AC^2 \text{ (Pyth)}$$

$$(15)^2 + (5)^2 = AC^2 \quad \checkmark^a \text{ substitute into correct formula}$$

$$AC^2 = 250$$

$$AC = 5\sqrt{10} \quad \checkmark^{ca} \text{ simplification}$$

$$\text{Perimeter} = 15 + 5 + 5\sqrt{10} = 20 + 5\sqrt{10} \quad \checkmark \checkmark^{ca} \text{ answer}$$

(6)

[12]

QUESTION 2

2.1 2.1.1 $r^2 = x^2 + y^2$

$$r^2 = 8^2 + 4^2$$

$$r^2 = 80 \checkmark$$

$$\therefore x^2 + y^2 = 80 \checkmark \quad (2)$$

2.1.2 $A(-8; -4) \checkmark \checkmark \quad (2)$

2.1.3 $A(-8; -4) \quad B(8; 4)$

$$m_{AB} = \frac{y_A - y_B}{x_A - x_B} = \frac{-4 - 4}{-8 - 8} = \frac{1}{2} \checkmark$$

$$\therefore y = \frac{1}{2}x \checkmark \quad (2)$$

2.1.4 $m_{PT} \times m_{AQ} = -1 \quad (\tan \perp \text{rad})$

$$\therefore m_{PT} = -2 \checkmark$$

$$\tan \theta = -2 \checkmark$$

$$\theta = 180^\circ - 63,43^\circ = 116,57^\circ \checkmark \quad (3)$$

2.1.5 $y = -2x + c \quad \text{OR}$

$$y - y_1 = m(x - x_1)$$

$$-4 = -2(-8) + c \checkmark$$

$$y + 4 = -2(x - 8) \checkmark \checkmark$$

$$c = -20 \checkmark$$

$$y = -2x - 20 \checkmark$$

$$\therefore y = -2x - 20 \checkmark \quad (3)$$

2.1.6 $y = -2x + -20$

$$m(AT) = -2$$

$$-10 = -2t + -20 \checkmark \quad \text{OR}$$

$$\frac{-4 + 10}{-8 - t} = -2$$

$$t = -5 \checkmark$$

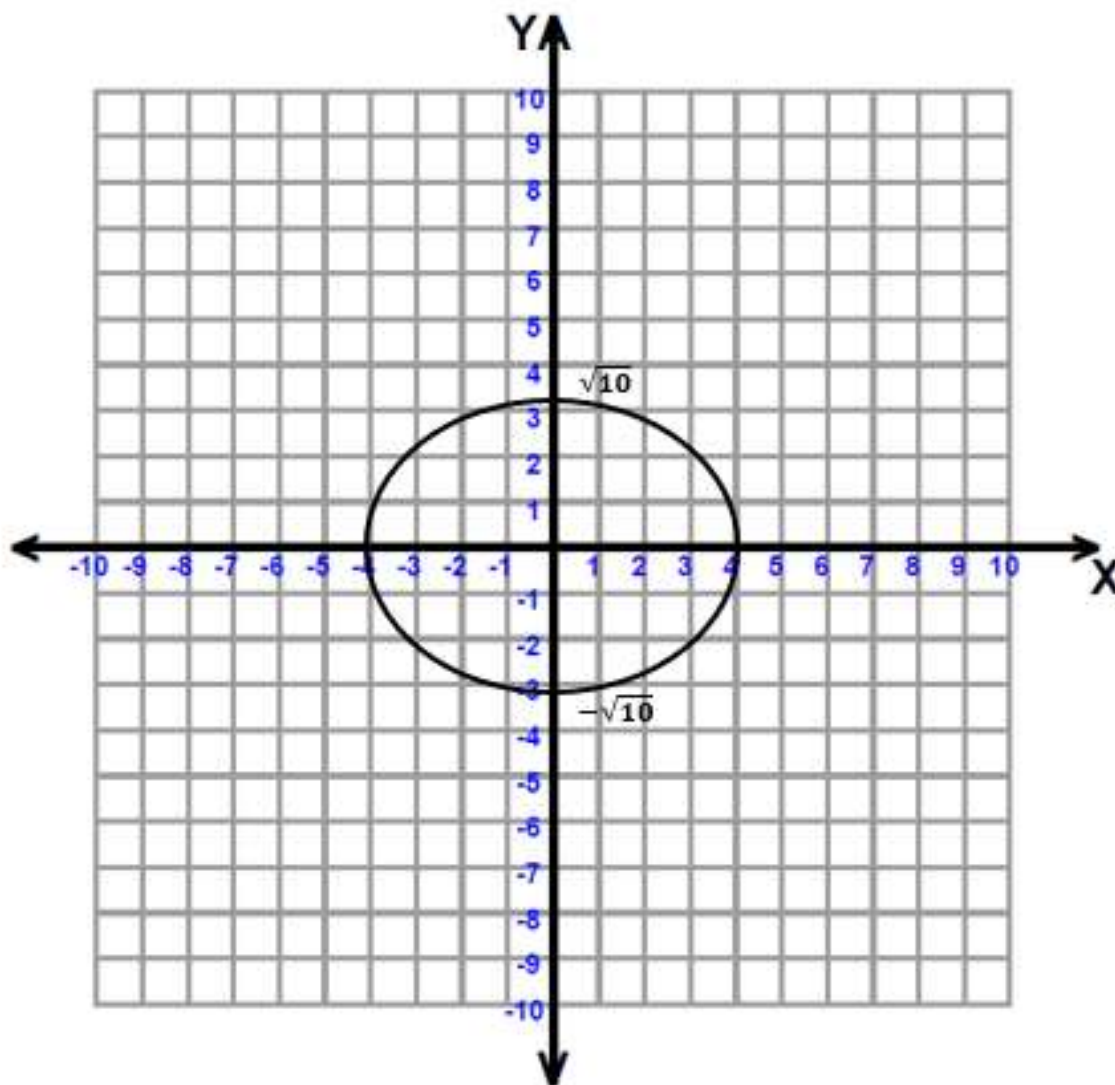
$$16 + 2t = 6$$

$$2t = -10$$

$$t = -5 \quad (2)$$

2.2 Sketch the graph defined by $\frac{x^2}{16} + \frac{y^2}{10} = 1$.

Clearly show ALL the intercepts with the axes.



- ✓ x-intercepts at 4 and -4
- ✓ y-intercepts at $\sqrt{10}$ and $-\sqrt{10}$
- ✓ shape

(3)

[17]

QUESTION 3

$$3.1 \quad 3.1.1 \quad = 6\left(\frac{4}{2\sqrt{13}}\right) \checkmark -3\left(\frac{4}{6}\right) \checkmark$$

$$= \frac{-26 + 12\sqrt{13}}{13} \checkmark$$

$$x^2 + y^2 = r^2$$

$$6^2 + 4^2 = AO^2$$

$$52 = AO^2$$

$$\therefore AO = 2\sqrt{13} \checkmark$$

(4)

$$3.1.2 \quad = \left(\frac{2\sqrt{13}}{4}\right)^2 \checkmark$$

$$= \frac{13}{16} \checkmark$$

(2)

$$3.2 \quad 3.2.1 \quad = \operatorname{cosec}\left(\frac{\pi}{3} - \frac{\pi}{6}\right)$$

$$= \operatorname{cosec}\left(\frac{\pi}{6}\right) \checkmark$$

$$= \frac{1}{\sin\left(\frac{\pi}{6}\right)}$$

$$= 2 \checkmark$$

(2)

$$3.2.2 \quad = 2 \cot\left(\frac{\pi}{6}\right) \checkmark$$

$$= \frac{2}{\tan \frac{\pi}{6}}$$

$$= \frac{2}{\frac{1}{\sqrt{3}}} \checkmark$$

$$= 2\sqrt{3} \quad \text{OR} \quad \approx 3,5$$

(2)

$$3.3 \quad 3.3.1 \quad 124,66^\circ \times \frac{\pi}{180^\circ} \approx 2,18 \text{ radians } \checkmark$$

$$57,46^\circ \times \frac{\pi}{180^\circ} \approx 1 \text{ radian } \checkmark \quad (2)$$

$$3.3.2 \quad \sec(2,18+1)$$

$$= \sec(3,18)$$

$$= \frac{1}{\cos(3,18)}$$

$$\approx -1,0 \checkmark \quad (1)$$

$$3.4 \quad \tan(180^\circ - \alpha) \cdot \cos \alpha \cdot \sin(180^\circ + \alpha) + \cos^2(360^\circ + \alpha)$$

$$= (-\tan \alpha) \checkmark (\cos \alpha) (-\sin \alpha) \checkmark + \cos^2 \alpha \checkmark$$

$$= (-\sin \alpha)(-\sin \alpha) + \cos^2 \alpha \checkmark$$

$$= \sin^2 \alpha + \cos^2 \alpha \checkmark$$

$$= 1 \checkmark \quad (6)$$

$$3.5 \quad \tan(x - 15^\circ) = -1$$

$$\therefore (x - 15^\circ) \checkmark = 180^\circ - 45^\circ \quad [\text{reference angle } \checkmark (x - 15^\circ)]$$

$$\therefore x = 150^\circ \checkmark \quad (3)$$

$$3.6 \quad \text{LHS: } \sin^2 A + \tan^2 A + \cos^2 A \quad \text{RHS: } \sec^2 A$$

$$= 1 \checkmark + \tan^2 A$$

$$= \sec^2 A \checkmark$$

$$= \text{RHS} \quad (2)$$

$$3.7 \quad 3.7.1 \quad a = 3; \checkmark b = 1; \checkmark c = 0; \checkmark d = 2 \checkmark \quad (4)$$

$$3.7.2 \quad \text{Periodicity} = \frac{360^\circ}{3} \checkmark$$

$$\text{Periodicity} = 120^\circ \checkmark$$

(2)

[30]

QUESTION 4

$$4.1 \quad 4.1.1 \quad \frac{30\text{m}}{\sin 65^\circ} \checkmark = \frac{AB}{\sin 50^\circ} \checkmark$$

$$\frac{30\sin 50^\circ}{\sin 65^\circ} = AB \checkmark$$

$$AB \approx 25,36 \text{ m} \checkmark \quad (4)$$

$$4.1.2 \quad \cos 25^\circ \checkmark = \frac{AD}{AB}$$

$$\cos 25^\circ = \frac{AD}{25,36\text{m}} \checkmark$$

$$25,36 \times \cos 25^\circ = AD$$

$$AD \approx 22,98 \text{ m} \checkmark$$

OR

$$\frac{AD}{\sin 65^\circ} \checkmark = \frac{25,36}{\sin 90^\circ} \checkmark$$

$$AD = \frac{25,36 \times \sin 65^\circ}{\sin 90^\circ}$$

$$AD \approx 22,98 \text{ m} \checkmark$$

(3)

$$4.2 \quad 4.2.1 \quad AC^2 = AB^2 + CB^2 - 2AB.CB.\cos 60^\circ \checkmark$$

$$AC^2 = (680 \text{ m})^2 + (420 \text{ m})^2 - 2(680 \text{ m})(420 \text{ m})\cos 60^\circ \checkmark$$

$$AC^2 = 353200 \checkmark$$

$$AC \approx 594 \text{ m} \checkmark \quad (4)$$

$$4.2.2 \quad 120^\circ \checkmark \quad (1)$$

$$4.2.3 \quad \frac{AD}{\sin 41^\circ} = \frac{594 \text{ m}}{\sin 120^\circ} \checkmark$$

$$\therefore AD = \frac{594 \times \sin 41^\circ}{\sin 120^\circ} \checkmark$$

$$AD \approx 450 \text{ m} \checkmark \quad (3)$$

[15]

QUESTION 5

5.1 In $\triangle ABC$: $AC^2 = AB^2 + BC^2$ (Pyth)

$$AC^2 = (230)^2 + (230)^2 \checkmark$$

$$AC = 230\sqrt{2} \checkmark \quad \text{OR} \quad \therefore AC = 325,269\dots$$

$$\therefore MC = \frac{230\sqrt{2}}{2} \quad \text{OR} \quad \therefore MC = \frac{325,269}{2}$$

$$\therefore MC = 115\sqrt{2} \checkmark \quad \text{OR} \quad \therefore MC = 162,63 \quad (3)$$

5.2 $\cos \hat{MCT} = \frac{MC}{TC} \checkmark$

$$\cos \hat{MCT} = \frac{115\sqrt{2} \text{ m}}{218 \text{ m}} \checkmark$$

$$\therefore \cos \hat{MCT} = 0,746$$

$$\therefore \hat{MCT} \approx 41,75^\circ \checkmark \quad (3)$$

[6]

QUESTION 6

6.1 6.1.1 $\hat{H}_1 = \hat{F}_1 = 23^\circ$ ✓ (angles in same segment) ✓
 $\hat{F}_1 + \hat{F}_2 = \hat{E}_3 = 50^\circ$ (equal angles opp equal sides; radii) ✓
 $\therefore 23^\circ + a = 50^\circ$
 $a = 27^\circ$ ✓ (4)

6.1.2 $\hat{E}_1 = \hat{F}_2 + \hat{F}_3$ (angles in same segment) ✓
 $49^\circ = 27^\circ + b$
 $22^\circ = b$ ✓ (2)

6.2 6.2.1 $\hat{B}_2 + \hat{F} = 180^\circ$ (opp. angles of cyclic quad) ✓
 $\hat{B}_2 + 35^\circ = 180^\circ$
 $\therefore \hat{B}_2 = 145^\circ$ ✓ (2)

6.2.2 $\hat{E}_1 = 2 \times \hat{F}$ (angle at centre = 2 x angle at circumference) ✓
 $\hat{E}_1 = 2 \times 35^\circ = 70^\circ$ ✓ (2)

6.2.3 $\hat{E}_1 + \hat{C}_3 + \hat{D}_2 = 180^\circ$ (int. angles of triangle CED) ✓
 $70^\circ + \hat{C}_3 + \hat{D}_2 = 180^\circ$
 $\hat{C}_3 = \hat{D}_2$ (equal angles at equal sides; radii) ✓
 $\therefore \hat{C}_3 = \hat{D}_2 = 55^\circ$ ✓ (3)

6.2.4 $2 \hat{A} = \hat{B}_2$ (angle at centre = 2 x angle at circumference) ✓
 $2 \hat{A} = 145^\circ$
 $\therefore \hat{A} = 72,5^\circ$ ✓ (2)

6.2.5 $\hat{A} = \hat{C}_1 = 72,5^\circ$ (equal angles at equal sides; radii)
 $\hat{C}_1 + \hat{C}_2 = 90^\circ$ (angle in semi-circle) ✓
 $\therefore \hat{C}_2 = 17,5^\circ$ ✓ (3)

6.3 6.3.1 $\hat{A}_1 = \hat{C}_1$ ✓ (angles opposite equal sides, given $AD = DC$) ✓

$$\therefore \hat{A}_1 = 37,2^\circ \quad \checkmark \quad (2)$$

6.3.2 $\hat{A}_1 = \hat{C}$ (angles opposite equal sides, given $AD = DC$) ✓

$$\hat{A}_1 = \hat{B}_1 = 37,2^\circ \quad \checkmark \quad (\text{tan-chord theorem}) \quad \checkmark$$

$$\hat{B}_2 = 90^\circ - 37,2^\circ \quad \checkmark \quad (\text{angle in semi circle})$$

$$\therefore \hat{B}_2 = 52,8^\circ \quad \checkmark \quad (4)$$

6.3.3 $\hat{ABE} = 90^\circ$ (angle in semi-circle) ✓

$$AE^2 = BE^2 + AB^2 \quad (\text{Pythagoras})$$

$$(13)^2 = (3,5)^2 + AB^2 \quad \checkmark$$

$$\therefore AB = 12,52 \text{ cm} \quad \checkmark \quad (3)$$

6.4 6.4.1 $\frac{PB}{BR} = \frac{PC}{CA} = \frac{1}{2}$ ✓ (Proportionality theorem, $CB \parallel AR$)

$$\frac{PA}{PQ} = \frac{PC + CA}{PC + CA + AQ} \quad \checkmark$$

$$\frac{3}{8} = \frac{1+2}{1+2+5} \quad \checkmark$$

$$\therefore \frac{BD}{BQ} = \frac{CA}{CQ} \quad \checkmark$$

$$= \frac{2}{5} \quad \checkmark \quad (5)$$

$$\begin{aligned} 6.4.2 \quad & \frac{\text{Area of } \triangle PRA}{\text{Area of } \triangle QRA} \\ &= \frac{0,5 \times PA \times \text{height}}{0,5 \times QA \times \text{height}} \checkmark \\ &= \frac{PA}{QA} \\ &= \frac{3}{5} \checkmark \end{aligned}$$

(2)

[34]

QUESTION 7

$$7.1 \quad \text{Area of a sector} = \frac{rs}{2} = \frac{r^2\theta}{2}, \checkmark$$

r = radius, s = arc length and θ = central angle in radians

$$\text{Area} = \frac{(9 \text{ cm})^2 \left(80^\circ \times \frac{\pi}{180^\circ} \right)}{2} \checkmark$$

$$\text{Area} = 18\pi \text{ cm}^2 \checkmark$$

$$\text{Area of triangle AOC} = \frac{1}{2} \times AO \times OC \times \sin 80^\circ$$

$$\therefore \text{Area} = \frac{1}{2} \times 9 \text{ cm} \times 9 \text{ cm} \times \sin 80^\circ \checkmark$$

$$\therefore \text{Area} = 39,884714 \text{ cm}^2 \checkmark$$

$$\text{Area of shaded segment} = 18\pi \text{ cm}^2 - 39,884714 \text{ cm}^2$$

$$\approx 16,7 \text{ cm}^2 \checkmark \quad (6)$$

$$7.2 \quad 7.2.1 \quad v = \pi Dn$$

$$v = \pi(0,15)(4,2) \checkmark \checkmark$$

$$v = 1,98 \text{ m/s} \checkmark \checkmark \quad (4)$$

$$7.2.2 \quad \omega = 2\pi n \checkmark$$

$$\omega = 2\pi(4,2) \checkmark$$

$$\omega = 8,4\pi \text{ rad/s} \checkmark \quad (3)$$

$$7.2.3 \quad (a) \quad s_1 = r\theta$$

$$s_1 = (7,5) \left(160^\circ \times \frac{\pi}{180^\circ} \right) \checkmark$$

$$s_1 \approx 21 \text{ cm} \checkmark \quad (2)$$

$$(b) \quad s_2 = r\theta$$

$$s_2 = (15) \left(200^\circ \times \frac{\pi}{180^\circ} \right) \checkmark$$

$$s_2 \approx 52,4 \text{ cm} \checkmark \quad (2)$$

$$(c) \quad (52,5)^2 = (7,5)^2 + AB^2 \quad \checkmark$$

$$CD = AB \approx 52 \text{ cm} \quad \checkmark$$

$$\begin{aligned} \text{Total length} &= (51,96 + 51,96 + 20,94 + 52,36) \text{ cm} \quad \checkmark \\ &\approx 177,3 \text{ cm} \quad \checkmark \end{aligned}$$

(4)

[21]

QUESTION 8

8.1 Hemisphere $Volume = \frac{4}{3}\pi r^3 \div 2$

$$Volume = \frac{4}{3}\pi(20)^3 \div 2 \checkmark$$

$$= 16755,16 \text{ m}^3 \checkmark$$

$$\text{Cylinder Volume} = \pi r^2 \times h$$

$$= \pi(20)^2 \times 8 \checkmark$$

$$= 10053,1 \text{ m}^3 \checkmark$$

$$\text{Volume in one tank} = 16755,16 \text{ m}^3 + 10053,1 \text{ m}^3$$

$$= 26808,26 \text{ m}^3 \checkmark$$

$$\text{Total volume of fuel that the tanker can carry in the 4 tanks} = 4(26808,26 \text{ m}^3)$$

$$= 107233,04 \text{ m}^3 \checkmark$$

(6)

8.2 8.2.1 $a = 120 \text{ m} \div 6 = 20 \text{ m}$

$$Area = a(m_1 + m_2 + m_3 + m_4 + m_5) \checkmark$$

$$= 20 \left(\frac{12+15}{2} + \frac{15+16}{2} + \frac{16+17}{2} + \frac{17+17}{2} + \frac{17+16}{2} + \frac{16+12}{2} \right) \checkmark$$

$$= 20(13,5 + 15,5 + 16,5 + 17 + 16,5 + 14) \checkmark$$

$$= 1860 \text{ m}^2 \checkmark$$

(4)

8.2.2 $Volume = 1\,860 \text{ m}^2 \times 210 \text{ m} \checkmark$

$$= 390\,600 \text{ m}^3 \checkmark$$

(2)

8.2.3 $Volume \text{ for } 1 \text{ m deep} = 120 \text{ m} \times 210 \text{ m} \times 1 \text{ m} \checkmark$

$$= 25\,200 \text{ m}^3 \checkmark$$

$$1 \text{ m}^3 = 1\,000 \text{ litres}$$

$$25\,200 \text{ m}^3 = 25\,200\,000 \text{ litres} \checkmark$$

(3)

[15]**Total: 150 marks**